

# Influence of Multi Wall Carbon Nanotube(MWCNTs) Reinforcement on the Mechanical Properties and Vibration Behavior of Composite Plates

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**Abstract**—The Nano particles affect onto the mechanical properties of the materials. This effect leads to an effect on the vibration characteristic of plate structure like the natural frequency, mode shape, the response, and the induced stress in the plate. In this paper a study is done on the composite plate combined of woven reinforcement fiber and polyester resin materials, and then, the effect of Nano particle on to natural frequency and mechanical properties of the plate is investigated. The work included an evaluation of the natural frequency and the mechanical properties (modulus of elasticity) of the composite plate with different Nano particle weight fraction effect, where, the weight fraction studied are (0.25%, 0.5%, 0.75%, and 1%, from resin materials weight). An experimental study is used to evaluate the mechanical properties and natural frequency of composite materials plate with Nano particle effect, then, depending on this mechanical properties evaluated from the experimental work ; the natural frequency of plate is studied theoretically (analytically and numerically) to compare with the experimental results. A comparison of natural frequency results shows a good agreement between experimental and theoretical work, since the maximum error between its works about (8.73%). Finally, the results show that the Nano particle increasing the natural frequency and mechanical properties of plate materials with higher value at Nano particle weight fraction about (1%).

**Keywords**— Vibration Nano composite plate, Nano particle effect, Nano composite materials, mechanical properties with Nano.

## I. INTRODUCTION

Elaboration of specimens for composite plate reinforced with carbon fiber woven, the main specification of carbon fiber is a new material, and it has a clear advantage of high tensile strength, resistance of corrosion, perfect conductivity, and high temperature resistance. In addition to carbon woven fabric filled the matrix of composite with specific weight ratio of MWCNTs as illustrated in [1]. The Nanotechnology possible applications in several field, and many of researchers studied the effect of Nano particle on different parameters as,

Presented the influence of silica Nanoparticle on mechanical properties of silica-epoxy composite, which, the paper studied experimentally the effect of Nano size, shape and loading on mechanical behavior as tensile, compressive

and flexural behavior, [2]. After this, studied experimentally the effect of carbon Nanotube reinforcement on the mechanical behavior and temperature-viscosity profile of epoxy matrix, the work shows the reinforcement with Nanotube modification the mechanical properties of composite, [3]. Then, presented experimentally and theoretically work to effect of three dimension reinforcement of woven composite, the paper included measurement the inter-laminar fracture and tension bearing of composite, [4]. Also, studied the effect of Nanotubes on fracture and composite materials mechanical properties, the work presented the modified the mechanical and fracture behaviors of composite materials with reinforcement by Nanotubes carbon. In addition to the researchers studied the other properties as chemical properties and thermal properties, [5]. In addition to, many researchers presented the effect of sandwich structure on the mechanical and vibration behavior of plate structure, [6]. Also, presented the effect of powder reinforcement on mechanical properties and different behavior of composite plate structure, [7, 8], and shown that the reinforcement with powder is modified its properties.

But, the above researchers are not study the effect Nano particle on the dynamic behavior of composite plate. Therefore, in this research studying the effect for weight ratio of carbon Nano tubes on the natural frequency of plate (with different boundary condition of plate) and composite materials mechanical properties by theoretical (analytical and numerical) and experimental work. Also, comparison the results with it to shown the agreement between the investigation work are used.

## II. EXPERIMENTAL WORK

The experimental work contains an investigation of the effect of the Nano particles on the mechanical properties of the composite plate and on its fundamental natural frequency. The first step was the manufacturing of the tested samples of the mechanical properties and of the vibration test. The second was applying these tests on the manufactured samples.

In this study, woven glass fiber fabric type plain weave is used depending on the required woven with highest properties

of lightness of weight and high modulus of elasticity. A real weight per unit area of woven glass fabric is  $300 \text{ g/m}^2 \pm 15 \text{ g/m}^2$  at width fabric design thickness 0.166 mm.

The actual densities of the glass fiber fabric and resin are computed by take rectangular sheets ( $20 \text{ cm} \times 17 \text{ cm}$ ) from fiber glass, and afterward a rectangular sheet of fiber was weighted. The volume of the rectangular sheet was measured using runway beaker filled with water incompletely, and put the fiber in the beaker. Therefore, the volume of fiber put in the beaker is equal to the increasing for the volume of water after putting of the fiber. The density of fiber can be computed by the following formula:

$$\rho_o = \frac{\text{Weight}}{V_o (\text{Increasing of water volume})} \quad (1)$$

The density of resin can be measured utilizing a weighted volume. The density measurement of resin was observed to be ( $1200 \text{ kg/m}^3$ ) and the density of glass fiber ( $2300 \text{ kg/m}^3$ ).

To satisfy vacuum bag laminating process needed to embossing templates gypsum material with dimensions ( $22 \text{ cm} \times 22 \text{ cm}$ ). The gypsum template is placed on the metallic vacuum pipe, and then a gypsum template was inserted in a bag of PVA to isolate gypsum of template from the interaction between woven carbon sheets and resin. Insert the bag in to the vacuum process by close it tightly from top and bottom. The template then coated by a layer with ( $26 \text{ cm} \times 26 \text{ cm}$ ) of woven glass fabric dry rectangular sheets As shown in Fig. 1, so the remainder of the woven glass fabric rolled into the back of the mold and governs at the back with adhesive tape. This process is repeated for each layer until the staking of woven carbon fiber sheets complete. Thereafter, another bag of PVA with the provisions of the closer from the top and bottom to avoid exit the resin. The resin was flowed inside the PVA bag from the top as shown in Fig. 1 , when vacuum process occurs. The vacuum device was switched on twice before coating glass sheets and after the resin mixed with hardener. It is still work until the resin pass through the woven glass fabric and begin the excess resin to exit from outlet pipe which is connected with vacuum device as well as the composite materials begin to harden. In this work to get at the end specimen as shown in Fig. 2, after fixing the specimen dimension, cutting with electrical cutter. These specimens of different weight percentage of MWCNTs are tested under vibration test.

#### A. Ultra-sonication process for reinforced MWCNTs

As an alternative sonicator, ultrasonic Homogenizer (300VT) is used in the present practical. It is multi-function and multi-purpose instruments that utilize strong super sound to induce cavitation effect in liquid, the instruments deal with the super sound to the material. Power supply turns the single-phase electricity of 220VAC, 50Hz into the alternating electricity of 20-25 kHz, 600V by frequency conversion device. Vibrational wave induces cavitation effect of various taking broken cells by titanium alloy variable amplitude lever which is immersed in the sample solution, to break cells.

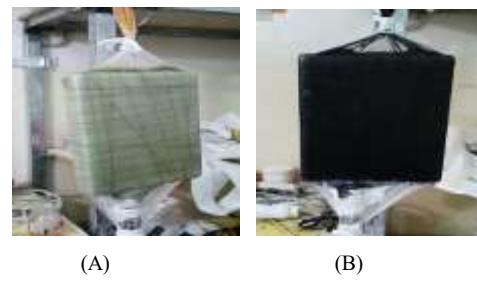


Fig. 1. Gypsum template coated with woven glass fabric with Rein inside of PVA a) without MWCNTs, b) with MWCNTs.

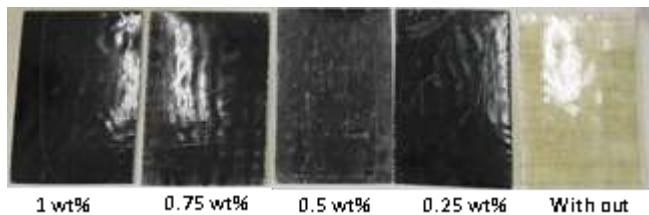


Fig. 2. Composite plate reinforced with glass fiber woven and plus wt% MWCNTs.

A range of ratios (0.25, 0.5, 0.75, and 1.0)wt% of Multi-Walled Carbon Nanotubes (MWCNTs), with diameter (6-9) nm and length of tube 5  $\mu\text{m}$  (O.D. = 6-9nm & L= 5  $\mu\text{m}$ ) as a filler in addition to woven fabric to reinforced the lamination 80:20 resin, then the mixture by Ultrasonic Homogenizer are mixing homogeneously, with equal period of processing time about 20-30 minutes the taking about 5-10 minutes for rest to avoid the rise of the mixture temperature to more than 80oc, and therefore the cooling water as in Fig.3.a, used to reduce the increasing in temperature of mixture due to break the bonds of resin by titanium probe in order to obtain a good dispersion of MWCNTs through the resin. The rapid vibration of the probe tip causes cavitation After 15 minutes, which formulated of black bubbles. The collapse of cavitation bubbles releases energy in the cavitation field which in turn increases the temperature of the mixture. It is not permitted to stop until they are sure of the homogeneity of the mixture.

#### B. Mechanical properties measurement

The mechanical properties of composite materials with Nano effect are measured using tensile test machine. The tensile test specimens were produced according to ASTM D 638M, [9], and are shown in Fig. 3.a which is type IV(44) ,it is used for composite materials. Where, the tensile specimens were tested using five samples for each weight ratio effect sample. Also, the dimensions of tensile composite materials with and without Nano sample are shown in Fig. 3.b It was used a tensile testing device , type LARYEE universal machine at a speed range ( 0.5mm/min- 5mm/min) with range of loads (1000KN) was utilized for the tensile test whose control and data acquisition are performed by a computer. The specimens were mounted in the grips of testing machine and they should be aligned to the longitudinal axis of the specimen coinciding with the direction of the applied load at speed 2mm/min according to ASTM D638 for composite materials as shown in Fig. 4.

The tension tests curves with and without (MWCNTs) were mostly nonlinear, and there was sudden fracture of the composite sample when the load is achieved a greatest value. It is evident that the initial slope of the curve and the maximum stress was uttermost for the composite with MWCNTs than those without, indicating increases modulus of elasticity of the composites due to MWCNTs. The elastic modulus of the composite material can be calculated using the stress-strain-carve obtained through the experimental test. The results of five samples take its average for different weight percentage of MWCNTs that added to composite. The modulus of elasticity results obtained from experimental tensile tests have been shown in Fig. 5, for five samples for each weight ratio Nano effect.

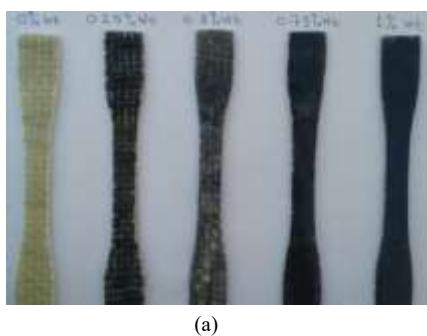


Fig. 3. Specimens and dimensions for tensile test.



Fig. 4. Specimen in machine (tensile test).

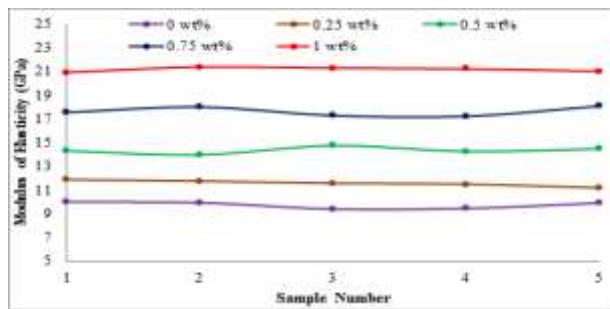


Fig. 5. Modulus of elasticity (Gpa) for five samples of each weight ratio effect at different MWCNTs.

### C. Natural frequency measurement

The vibration part evaluates the fundamental natural frequency of composite materials plate with Nano particle effect. The evaluation of natural frequency is investigated by testing the composite plate with vibration rig shown in Fig. 6. The compounds of vibration rig are, structure frame, digital storage oscilloscope, amplifier, impact hammer tool and accelerometer, [10, 11, 12 and 13]. The dimensions of plate sample used in this work are, length of plate  $a = 20 \text{ cm}$ , width of plate sample  $b = 15 \text{ cm}$ , and, thickness of plate sample  $h = 2 \text{ mm}$ . Therefore, the plates sample tested by vibration rig are supported with various supported as, shown example in Fig. 7, simply supported at all edges (SSSS); simply supported along two edges and free along other edges (SSFF); cantilever supported (CFFF); clamped supported at all edges (CCCC); simply supported along two edge and clamped along other edges (CCSS); and clamped supported along two edges and free along other edges (CCFF). After this, the signal evaluated by digital storage oscilloscope analysis by suing fast Fourier transformation method [10-13], by using sig-view program as in Fig. 8, to evaluated the frequency of plate.

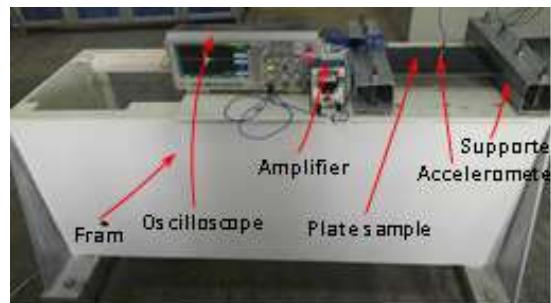


Fig. 6. Rig Component of Vibration Plate.



Fig. 7. Different Supported of Vibration Plate Sample.

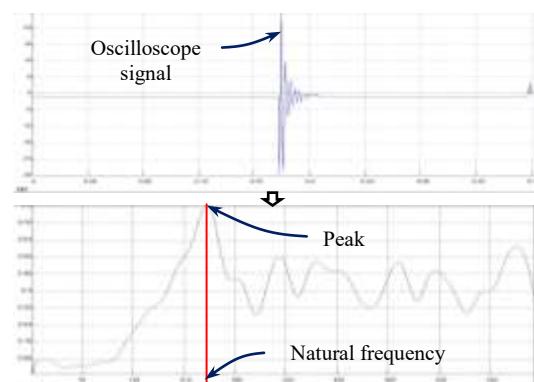


Fig. 8. Analysis of acceleration signal with FFT, to evaluating the frequency.

### III. THEORETICAL INVESTIGATION

The theoretical investigation included, first, evaluated the frequency of plate, supported as simply supported, with Nano effect by analytical solution of general equation of motion of orthotropic plate, and second, evaluated the frequency of plate with various supported by using finite element method with using of Ansys program, and then, comparing the results.

#### A. Analytical Work

The general equation of motion solution for orthotropic plate can be used to calculate the frequency of composite plate (reinforcement with woven fiber and effect of Nano particle), using equation, [14],

$$D_{11} \frac{\partial^4 w}{\partial x^4} + D_{22} \frac{\partial^4 w}{\partial y^4} + 2(D_{12} + 2D_{66}) \frac{\partial^4 w}{\partial x^2 \partial y^2} + \rho h \frac{\partial^2 w}{\partial t^2} = 0 \quad (2)$$

$$\text{Where, } D_{11} = \frac{E_1 h^3}{12(1-\nu_{12}\nu_{21})}, \quad D_{22} = \frac{E_2 h^3}{12(1-\nu_{12}\nu_{21})}, \quad D_{66} = \frac{G_{12} h^3}{12},$$

$$D_{12} = \frac{\nu_{12} E_2 h^3}{12(1-\nu_{12}\nu_{21})}, \quad G_{12} = 2.67 \text{ GPa}, \quad \nu_{12} = \nu_{21} = 0.33, \quad [15].$$

And  $E_1, E_2$ , are modulus of elasticity in 1 and 2-direction, respectively, and  $\rho$  is the density of composite materials plate, using the values evaluated from experimental work. Where, the density can be calculated from Eq. 1, for 52% volume fraction of woven fiber reinforcement and 48% volume fraction of polyester resin materials, therefore,  $\rho=1772 \text{ kg/m}^3$ .

The solution of Eq. 2 is dependent on the boundary condition of plate, then, for simply supported plate, substitution the general behavior of plate ( $w(x, y, t) = A \sin \frac{m\pi x}{a} \cdot \sin \frac{n\pi y}{b} \cdot \sin \omega t$ ), [16], in to Eq. 2, get the equation given the natural frequency plate with effect of Nano and reinforcement fiber, as, [14, 16],

$$\omega_{mn}^2 = \frac{\pi^4}{\rho h a^4} \left[ D_{11} m^4 + D_{22} n^4 \left( \frac{a}{b} \right)^4 + 2(D_{12} + 2D_{66}) m^2 n^2 \left( \frac{a}{b} \right)^2 \right] \quad (3)$$

Where,  $m$  and  $n$  are integers, and equal one for fundamental natural frequency of plate. Therefore, the analytical results of natural frequencies are compared with experimental and numerical results for simply supported plate.

#### IV. NUMERICAL WORK

The numerical techniques, such as finite element method, boundary element method are widely used in solving the modal problems. In the FEM, every individual from the structure is subdivided into a progression of genuinely short element, each connected to each other at the nodal points. The choice of element relies on upon the physical behavior of the body under real loading conditions and on how near the real conduct the expert needs the outcomes to be. ANSYS gives an assortment of element type starting from one dimensional to three dimensional element. This method has a number of advantages that have made it very popular [17]. For the accomplish of the present work, a shell 281 element is chosen which is appropriate for examining thin to modestly thick shell structures. The components contain eight nodes with six DOF at every node: rotations about the x, y, and z axes, and translations in the x, y, and z axes. It might be

utilized for layered applications for demonstrating composite shells. It is incorporate the influence of transverse shear deformation. The Fig. 9.A demonstrates the geometry, the element coordinate, and the nodes locations for this element. The element is characterized by eight nodes (I, J, K, L, M, N, O and P) and shell section properties [16]. Shell 281 can be related with creep, elasto-plastic, or hyperplastic material properties. Just orthotropic linear elastic, isotropic, anisotropic properties can be contribution for elasticity. The numerical investigation of several orthotropic composite plate are assessed by utilizing FEM used by the commercial program (ANSYS. ver. 15).A linear elastic orthotropic was performed to compute the natural frequencies and mode shapes of the structure. FEM model is accomplished by using APDL ANSYS program with SHELL 281.

The natural frequencies results evaluated by ANSYS program Ver. 15 are comparing with experimental results with different weight fraction of MWCNTs in composite plate. Meshing is done by taking a grid of 50X30 shell 281 elements as shown in Fig. 9.B. The mechanical properties used as the input data of ANSYS are the modulus of elasticity, density, evaluated from the experimental work, and other properties which had been estimated from the theoretical results, are presented in [15]. In addition, the natural frequency evaluated numerically are compared with the experimental and analytical results calculated for different supported.

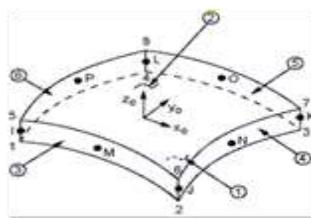


Fig. 9. A. Shell 281 Geometry.

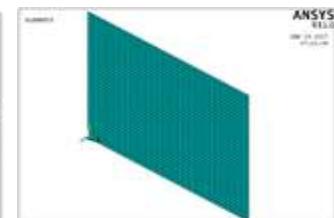


Fig. 9. B. Meshing of plate

#### V. RESULTS AND DISCUSSION

The effect of MWCNTs on the frequency of composite plate, composed of 52% volume fraction of reinforcement woven fiber and 48% volume fraction of polyester resin materials is presented using three techniques, first is experimentally, second is analytically and third is numerically investigation. The experimental technique measures the frequency and mechanical properties (modulus of elasticity) of composite plate with different boundary conditions of fixing and various MWCNTs weight ratio effect, where, the Table 1 shows the modulus of elasticity of composite materials with various MWCNTs. From this table it can be seen that the addition of MWCNTs modifies the properties of the material. Also, the fundamental frequency of simply supported plate is calculated analytically using Eq. 3, where, the required mechanical properties of composite materials used in Eq. 3 are shown in Table 1 and other mechanical properties is presented by Eq. 1 and [15]. After this, the natural frequency are evaluated numerically for various simply supported plates and comparison is done with the data evaluated using other methods.

Therefore, Fig. 10, gives the comparison of the experimental, analytical and numerical data for the frequency of simply supported plate with various MWCNTs weight ratio. From the figure it is shown that the maximum error between analytical and numerical method is not exceed (0.9%) and the error between experimental and theoretically (analytical or numerical investigation) about (8.73%). Also, the comparison between experimentally and numerically results of natural frequency are presented in Figs. 11; 12; 13; 14 and 15, for different boundary conditions of composite plate. Where these figures show a good agreement between the evaluated data since the maximum error is not exceed (9%). Where, the reasons of it percent error are, the error become in the manufacture of composite materials plate, the fixed of plate on the vibration rig, the accuracy in measuring experimental data from the machine, and the accuracy of machine used.

A comparison of the results (to show the agreement between the technique used to calculate the frequency of the tested plate with Nano effect) is presented numerically , as shown in Fig. 16, which shows the effect of Nano weight ratio on the frequency of plate with different boundary conditions of the plate. Fig. 16 shows that the increasing of Nano modifies the natural frequency of the plate, due to modifying the modulus of elasticity for composite materials. Also, the results show the dynamic behavior of the composite plate for different mode shapes, where Fig. 17 shows first two mode shapes of CCFF (as an example) composite plate with 1 %wt Nano effect. Finally, from the presented results, shown in Table I and Figs. 10 to 17, it can be seen that the Nano particle modified the modulus of elasticity for composite materials and dynamic behavior of plate and the best ratio of Nano particle used is 1 %wt.

TABLE I. MODULUS OF ELASTICITY OF COMPOSIE AT DIFFERENT AMOUNT OF MWCNTS

NO.	%Wt of MWCNT	Modulus of Elasticity $E_1=E_2$ (Gpa)
1	0	9.72
2	0.25	11.56
3	0.5	14.33
4	0.75	17.6
5	1	21.13

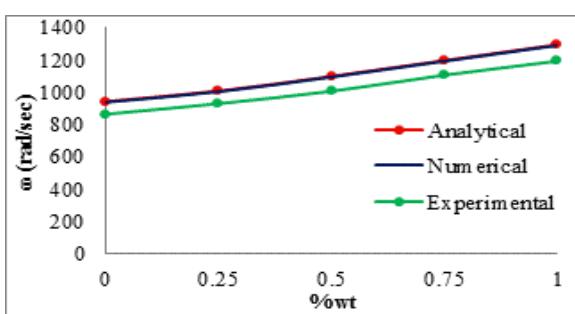


Fig. 10. Comparsion of natural frequency results for simply supported plate.

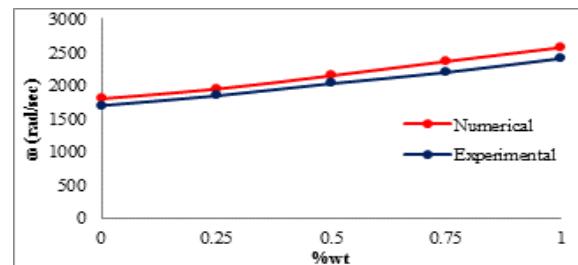


Fig. 11. Comparsion of natural frequency results for CCCC supported plate.

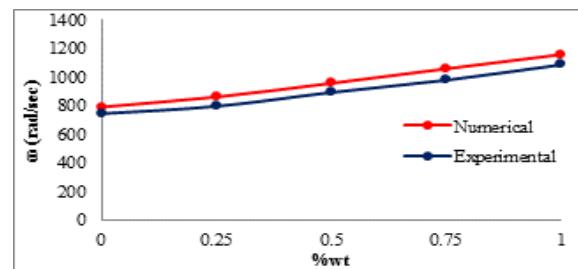


Fig. 12. Comparsion of natural frequency results for CCFF supported plate.

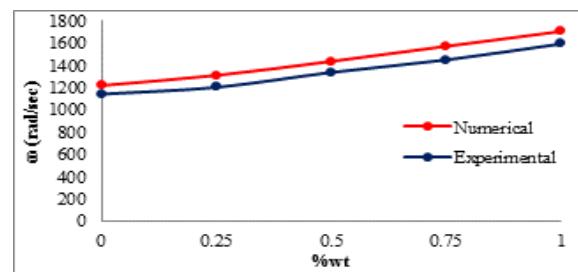


Fig. 13. Comparsion of natural frequency results for CCSS supported plate.

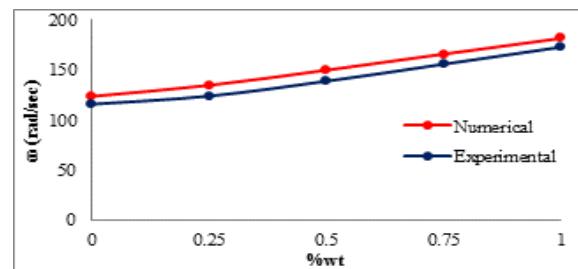


Fig. 14. Comparsion of natural frequency results for CFFF supported plate.

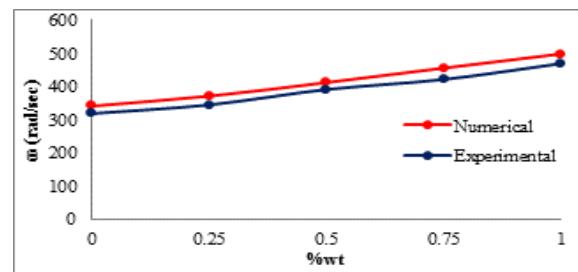


Fig. 15. Comparsion of natural frequency results for SSFF supported plate.

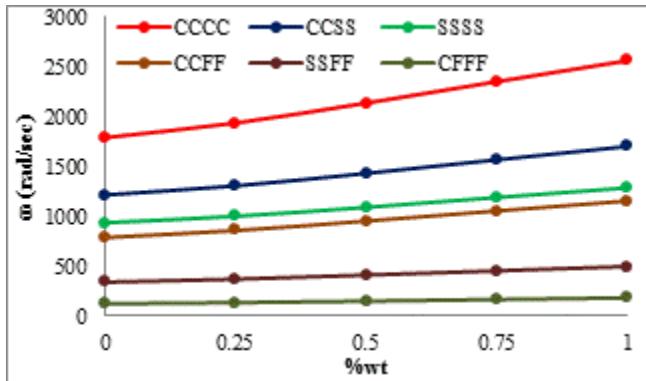


Fig. 16. Natural frequency of composite plate with Nano particle effect for various boundary condition plate.

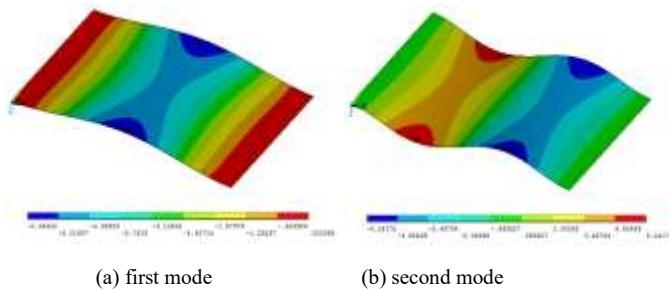


Fig. 17. Mode shape of CCFF composite plate.

## VI. CONCLUSION

From the presented work experimentally and theoretically, it can be concluded same points about the effect of Nano particle on the mechanical and dynamic behavior of composite plate structure, as below,

- The experimental work is powerful tool for measurements the mechanical properties and vibration behavior of composite materials plate structure reinforced with Nano particle.
- The Nano particle modify the natural frequency and the mechanical properties of composite materials for plate with high values comparison with increase of reinforcement woven fiber or powder reinforcement.
- The 1 wt% it is the best ratio can be used to modify the mechanical and vibration behavior of composite structures. Since, the mechanical properties and vibration behavior increasing at clear form at it ratio. But, in other ratio, less than or more than 1 wt%, mechanical properties and vibration behavior are worse.
- The comparison between experimental and theoretical (analytical or numerical) work shows a good agreement with maximum error about (9%).
- The comparison between analytical and numerical work, for simply supported plate, shows a good agreement with maximum error not exceed (0.9%).

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